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# Implications of a Massless Neutralino for Neutrino Physics

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## Abstract

We consider the phenomenological implications of a soft SUSY breaking term  $\tilde{B}N$  at the TeV scale (here  $\tilde{B}$  is the  $U(1)_Y$  gaugino and  $N$  is the right-handed neutrino field). In models with a massless (or nearly massless) neutralino, such a term will give rise through the see-saw mechanism to new contributions to the mass matrix of the light neutrinos. We treat the massless neutralino as an (almost) sterile neutrino and find that its mass depends on the square of the soft SUSY breaking scale, with interesting consequences for neutrino physics. We also show that, although it requires fine-tuning, a massless neutralino in the MSSM or NMSSM is not experimentally excluded. The implications of this scenario for neutrino physics are discussed.

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# 1 Introduction

The atmospheric neutrino data [1] gives convincing evidence of non-zero neutrino masses. This data also implies maximal or close to maximal mixing of muon and tau neutrino. Furthermore, the solar neutrino data [2] can also be naturally explained by non-zero mass splitting and mixing between the electron and muon neutrino. Recent data favors large mixing in this sector as well.

The question arises then, what is the physics behind neutrino masses and mixing patterns? What is the mechanism of neutrino mass generation? Why is the lepton mixing large, in contrast to the small mixing in the quark sector? These are among the most challenging problems of fundamental physics today.

Many mechanisms for neutrino mass generation have been suggested so far. Among these, the see-saw mechanism [3] seems to be the simplest and most natural. In this context, large neutrino mixing can appear due to large mixing in the charged lepton mass matrix [4] or in the Dirac mass matrix of neutrinos [5]. It can also appear from large mixing or very strong hierarchy in the Majorana mass matrix of the right handed neutrinos [6]. Large mixing in the lepton sector can also be obtained by radiative corrections due to renormalization group effects in schemes with quasi-degenerate neutrinos [7].

Models with three light neutrinos can explain the solar and atmospheric neutrino oscillation data, and also all laboratory neutrino experiments results, with the exception of LSND [8]. The explanation of the LSND experimental results requires either a fourth light neutrino [9, 10] (which, to satisfy the constraints coming from Z physics, has to be sterile), or violation of CPT in the neutrino sector [11]. In this later case, the neutrino and antineutrino masses can be different, thus providing an elegant solution to the LSND question. However, violation of CPT may be hard to accommodate theoretically. In the sterile neutrino case, one of the problems is that there is no compelling reason for the existence of such a particle. Also, even if the sterile neutrino is introduced by hand, it is hard to find a reason why it is so light, with mass of the order 1 eV, as required to explain LSND.

In this work we propose a scenario in which the sterile neutrino is an essentially massless (mass of order eV or less) bino. It can be shown that, in the framework of the general Minimal Supersymmetric Standard Model (MSSM), such a massless neutralino is still allowed experimentally. In order to couple this neutralino to the neutrino sector, we consider a new soft SUSY breaking term of the form  $c\tilde{B}N$ , where  $N$  is the right handed neutrino and  $c$  is a soft SUSY breaking mass parameter. This coupling will give the bino mass through the see-saw mechanism:  $m_{\nu_0} \sim c^2/M_M$ , thereby relating the mass of the sterile neutrino to the soft SUSY breaking scale. Moreover, the see-saw induced couplings of this bino with the three SM neutrinos naturally leads to large mixing between the three active families.

The paper is organized as follows. In sect. II we show how a massless neutralino can become the sterile neutrino. We also discuss the connection between the neutrino mass and the SUSY breaking scale, and how large mixing arises naturally in this model. In sect. III we formulate and review the conditions under which the lightest neutralino can be massless (both in MSSM and in NMSSM), and satisfy all the existing experimental constraints. A new  $U(1)_R$  symmetry is introduced in order to allow the bino-neutrino (or singlino-neutrino, in NMSSM) couplings, while forbidding all the usual R-parity violating (RPV) terms. In

sect. IV we consider the implications of our model for neutrino phenomenology in somewhat more detail. Conclusions are summarized in sect. V.

## 2 See-saw mechanism and the massless neutralino

In this section we will assume the existence of a massless neutralino, leaving the justification for this assumption for later. This neutralino can be thought of as a superposition of higgsino and gaugino states. In the next section we will see that in the MSSM it has to be mostly bino in order to satisfy the existing experimental constraints.

Let us consider scenarios under which mixing between this state and the three SM neutrinos can arise. The first possibility is direct mixing: the higgsino states can couple to the neutrinos through bilinear RPV couplings  $\mu_i$ ; we will not study this scenario here. We shall consider the more interesting case when the mixing with the three SM neutrinos is obtained through coupling of the massless neutralino to the right-handed neutrino and the see-saw mechanism.

We take the right handed neutrino fields  $N_i$  to be singlets under the  $SU(2)_L \times U(1)_Y$  gauge group. Then they can couple only with the bino component of the massless neutralino, and to  $SU(2)_L \times U(1)_Y$  singlet components of the neutralino, if present. The corresponding bilinear couplings to the bino:

$$\tilde{B}N_i \quad (1)$$

have dimension three, and are therefore soft SUSY breaking terms [12]. The coefficients with which these terms appears in the Lagrangian (let's call them  $c_i$ ) will have dimensions of mass, and their magnitude is expected to be of the order of SUSY breaking scale, that is,  $\mathcal{O}(\text{TeV})$ .

The Lagrangian for the neutrino sector of our model will then be:

$$L = (m_D)_{ij}\bar{\nu}_i N_j + (M_M)_{ij}N_i N_j + c_i N_i \chi_0 + h.c., \quad (2)$$

where  $m_D$  and  $M_M$  are the Dirac and Majorana mass matrices for the three SM neutrinos, and  $\chi_0$  is the massless neutralino. We treat the massless neutralino as an (almost) sterile neutrino. Upon decoupling the heavy neutrino states (through the see-saw approximation) the mass matrix for the remaining four light neutrinos takes the form:

$$M_\nu = \begin{pmatrix} cM_M^{-1}c^T & cM_M^{-1}m_D^T \\ m_D M_M^{-1}c^T & m_D M_M^{-1}m_D^T \end{pmatrix} \quad (3)$$

where the first line corresponds to the sterile neutrino.

For simplicity of presentation let's assume in the following that the right handed neutrino mass matrix is diagonal and proportional to the identity matrix:  $(M_M)_{ij} = m_M \delta_{ij}$ . It can easily be seen then that the 4-neutrino mass matrix above has a zero eigenvalue, two eigenvalues of order  $m_D^2/m_M$  and one eigenvalue of order  $(c^2 + m_D^2)/m_M$ . The first three eigenvectors can be identified with the three SM neutrino mass eigenstates. The fourth eigenvector (which is mostly  $\chi_0$ ) can be identified with the sterile neutrino.

Note that, since the magnitude of the  $c$  terms is of order of soft SUSY breaking scale, they are naturally about ten times larger than the Dirac mass terms appearing in  $m_D$  (which

are of the order of the electroweak breaking scale, *i.e.*  $\mathcal{O}(100 \text{ GeV})$ . That means that the mass of the sterile neutrino will be about two orders of magnitude above the mass of the heavier SM neutrino, which, to account for the atmospheric neutrino data, has to be of order  $5 \times 10^{-2} \text{ eV}$ . This makes the mass of the sterile neutrino of eV size, which is the right value to explain the LSND experiment results. Moreover, we will show in section IV that Eq. (3) predicts the right value of mixing between the sterile and active neutrinos as well.

Notwithstanding the LSND experiment, the mass matrix in Eq. (3) with  $|c| \gg |m_D|$  naturally gives large mixing between the three light neutrinos. To see this we can decouple the sterile neutrino from the other three by using the see-saw approximation, and the mass matrix becomes:

$$M'_\nu = \frac{1}{m_M} \begin{pmatrix} cc^T & 0 \\ 0 & (m_D m_D^T)_{ij} - \frac{(cm_D^T)_i (cm_D^T)_j}{cc^T} \end{pmatrix}. \quad (4)$$

From this expression it can be seen that even if  $m_D$  is diagonal, we get off-diagonal entries of the same magnitude as the diagonal elements in the three SM neutrino mass matrix. This means that at least one mixing angle is large. Of course, in order to obtain the specific pattern of two large mixing angles and a small one, further conditions must be imposed. We will study this further in chapter IV.

We end this section with some comments. Above, we have assumed that the sterile neutrino is a massless neutralino. This does not necessarily have to be so. What is needed for the above see-saw scenario to work is a massless fermion, which is (or contains among its components) a singlet under the  $SU(2)_L \times U(1)_Y$  gauge group. Then, this fermion can couple with the right handed neutrinos through the soft SUSY breaking terms (1), and the mechanism presented above works. In the next section we will actually consider the case when the sterile neutrino is the NMSSM singlino. Some other SUSY particles (like a goldstino, which has the advantage of being naturally massless) can also play this role.

### 3 Massless Neutralino

In this section we will explore the possibility that supersymmetry allows the existence of a nearly massless neutralino. For the MSSM and the NMSSM, we show that a massless neutralino can be obtained by a fine-tuning of the soft breaking parameters. While we do not provide a reason for such a tuning, we do verify that the resulting massless neutralino is not yet excluded by experiment. By extending the visible sector particle content beyond that of the MSSM or NMSSM, it may be possible to achieve TeV scale visible sector SUSY breaking in a phenomenologically viable way. In such a case the lightest neutralino can be the goldstino and thus naturally massless, up to supergravity corrections of order  $\text{TeV}^2/M_{\text{Planck}} \sim 10^{-4} \text{ eV}$ . This is an interesting direction for further study.

In this section we will explore the possibility that supersymmetry allows the existence of a massless neutralino. Note that we will not try to give a reason why there should be such a particle (at this point, we do not know), but we will just simply verify that the existence of a massless neutralino is compatible with the experimental results from SUSY

searches. We shall consider two SUSY models, first the MSSM, and then the next-to-minimal supersymmetric model (NMSSM), which contains an extra singlet.

### 3.1 MSSM case

In the MSSM we have the following mass matrix for neutralinos:

$$M_{ij} \equiv \begin{pmatrix} -M_1 & 0 & -m_z \cos \beta \sin \theta_w & m_z \sin \beta \sin \theta_w \\ 0 & -M_2 & m_z \cos \beta \cos \theta_w & -m_z \sin \beta \cos \theta_w \\ -m_z \cos \beta \sin \theta_w & m_z \cos \beta \cos \theta_w & 0 & -\mu \\ m_z \sin \beta \sin \theta_w & -m_z \sin \beta \cos \theta_w & -\mu & 0 \end{pmatrix} \quad (5)$$

where  $M_1$  and  $M_2$  are the soft SUSY breaking mass terms for the  $U(1)_Y$  and  $SU(2)_w$  gaugino fields,  $m_z$  is the mass of the Z-boson,  $\tan \beta \equiv v_1/v_2$  and  $\mu$  is the Higgs mixing term.

The existence of a massless neutralino requires that the determinant of the mass matrix in (5) be zero:

$$\Delta_0 \equiv \mu m_z^2 \sin 2\beta (M_1 \cos^2 \theta_w + M_2 \sin^2 \theta_w) - \mu^2 M_1 M_2 = 0 \quad (6)$$

Most studies of the MSSM have been performed with the assumption of universal gaugino masses, where  $M_1$  and  $M_2$  are related by the GUT relation:

$$M_2 = \frac{5}{3} \tan^2 \theta_w M_1 \quad (7)$$

In this framework, Eq. (6) requires that:

$$\mu M_2 = \frac{m_z^2}{r} \sin 2\beta (r \cos^2 \theta_w + \sin^2 \theta_w) \quad (8)$$

with  $r = M_1/M_2 \simeq 0.5$ , which implies either that both  $\mu$  and  $M_2$  are of order  $m_z$ , or one of these parameters are much smaller than  $m_z$ . However, since  $\mu$  and  $m_2$  are responsible for the masses of the other neutralinos, as well as for the masses of the charginos, this can bring us into conflict with direct searches for these particles at LEP. A detailed analysis [14] shows that a massless neutralino is excluded in the MSSM with the GUT relation (7), except for a narrow region in the parameter space, where  $\tan \beta \simeq 1$ . However, this value for  $\tan \beta$  is excluded from other considerations.

In the search for a massless neutralino we are therefore led to give up the assumption of universal gaugino masses. Then, values for the  $M_1$  parameter:

$$M_1 = \frac{M_2 m_z^2 \sin 2\beta \sin^2 \theta_w}{\mu M_2 - m_z^2 \sin 2\beta \cos^2 \theta_w} \simeq \frac{m_z^2}{\mu} \sin 2\beta \sin^2 \theta_w \quad (9)$$

of order of a few GeV (or even smaller, for large  $\tan \beta$ ), can satisfy Eq. (6).  $M_2$  and  $\mu$  can be chosen sufficiently large to satisfy the constraints coming from  $Z$  decay to  $\chi^+ \chi^-$  and  $\chi_i^0 \chi_j^0$ , with  $i$  and  $j$  not 1 at the same time. One more constraint we have to consider comes from

the massless neutralino, which will give contribution to the invisible  $Z$  width. The current experimental [15] value:

$$\Gamma_Z^{inv} = 499.0 \pm 1.5 \text{ MeV} \quad (10)$$

requires that the branching ratio of  $Z$  to the massless neutralino pair be smaller than about 0.3 %:

$$\text{Br.}(Z \rightarrow \chi_1^0 \chi_1^0) < 3 \times 10^{-3}. \quad (11)$$

In order to figure out this branching ratio, we need to evaluate the particle content of the lightest neutralino and its interactions. If we define the mass eigenstates of the neutralino matrix by:

$$\chi_i^0 = N_{ij} \psi_j \quad , \quad \text{where } \psi_j = \{\tilde{B}, \tilde{W}^3, \tilde{h}^0, \tilde{h}^{\pm}\}, \quad (12)$$

then

$$N_{1i} = \left( 1, \frac{m_z^2 \sin 2\beta \sin 2\theta_W / 2}{m_z^2 \sin 2\beta \cos^2 \theta_W - \mu M_2}, -\frac{m_z M_2 \sin \beta \sin \theta_W}{m_z^2 \sin 2\beta \cos^2 \theta_W - \mu M_2}, \frac{m_z M_2 \cos \beta \sin \theta_W}{m_z^2 \sin 2\beta \cos^2 \theta_W - \mu M_2} \right) \quad (13)$$

up to a normalization constant. If  $\mu \gg m_z$ , that normalization constant is 1, and the massless neutralino is mostly bino. The interaction Lagrangian with the  $Z$  boson in terms of physical states is the following [13]:

$$\mathcal{L}_{Z\chi_i\chi_j} = \left( \frac{g}{2 \cos \theta_w} \right) Z_\mu \bar{\chi}_i \gamma^\mu (O_{ij}^L P_L + O_{ij}^R P_R) \chi_j \quad (14)$$

where

$$O_{ij}^L = \frac{1}{2}(-N_{i3}N_{j3}^* + N_{i4}N_{j4}^*), \quad O_{ij}^R = -O_{ij}^{L*}. \quad (15)$$

Therefore, for the massless neutralino we have:

$$O_{11}^L \simeq \frac{1}{2} \frac{m_Z^2}{\mu^2} \sin^2 \theta_W \cos 2\beta. \quad (16)$$

Eq. (11) requires that  $O_{11}^L \leq 1/30$ ; we see that this can be easily satisfied for values of  $\mu$  of order 500 GeV. (Note also that although in the limit  $\tan \beta \approx 1$  the massless neutralino is decoupled from  $Z$ -boson this is not a necessary condition).

So far we have shown that a massless neutralino is consistent with experimental constraints (if we give up the assumption of gaugino mass universality at GUT scale). This massless state, however, does not appear naturally in the theory; fine tuning of order 1 eV/100 GeV  $\simeq 10^{-11}$  is necessary to satisfy Eq. (6). Moreover, we have to impose this fine tuning on the complete theory; for example, if Eq. (6) holds at tree level, it will be broken by loop corrections, and the neutralino will acquire GeV size mass. For these reasons, this model is not very compelling theoretically; however, it is experimentally allowed.

### 3.2 NMSSM case

In the event we want to keep the GUT relation (7), it is possible to get a massless neutralino in the framework of NMSSM. Let us considering NMSSM with the following terms in the

superpotential:

$$W = \lambda \epsilon_{ij} H_1^i H_2^j S - \frac{1}{3} \kappa S^3 \quad (17)$$

where  $i, j = 1, 2$ ,  $\epsilon_{ij}$  is the antisymmetric tensor,  $H_1$  and  $H_2$  are the standard Higgs doublets and  $S$  is the MSSM Higgs singlet. (The first term replaces the usual SUSY  $\mu$  term). In this case the neutralino mass matrix becomes:

$$M_{ij} \equiv \begin{pmatrix} -M_1 & 0 & -m_z \cos \beta \sin \theta_w & m_z \sin \beta \sin \theta_w & 0 \\ 0 & -M_2 & m_z \cos \beta \cos \theta_w & -m_z \sin \beta \cos \theta_w & 0 \\ -m_z \cos \beta \sin \theta_w & m_z \cos \beta \cos \theta_w & 0 & \lambda x & \lambda v \sin \beta \\ m_z \sin \beta \sin \theta_w & -m_z \sin \beta \cos \theta_w & \lambda x & 0 & \lambda v \cos \beta \\ 0 & 0 & \lambda v \sin \beta & \lambda v \cos \beta & -2\kappa x \end{pmatrix}$$

The determinant of this mass matrix is:

$$\Delta = -2\kappa x \Delta_0 + \lambda^2 v^2 (m_z^2 (M_1 \cos^2 \theta_w + M_2 \sin^2 \theta_w) - \mu M_1 M_2 \sin 2\beta) \quad (18)$$

were  $v=174$  GeV,  $x$  is vacuum expectation value (VEV) of  $S$  field, and we define  $\mu$  to be  $\lambda x$ . We assume in the following that we are in a region of the parameter space where  $\mu M_1 M_2 \gg m_z^2 (M_1 \cos^2 \theta_w + M_2 \sin^2 \theta_w) \simeq 0.6 m_z^2 M_2$ ; we also assume that  $\tan \beta$  is large. Then  $\Delta_0 \simeq -\mu^2 M_1 M_2$ , and:

$$\kappa = \lambda \frac{1}{2} \left( \frac{\lambda v}{\mu} \right)^2 \frac{0.6 m_z^2 M_2 - 0.5 \mu M_2^2 \sin 2\beta}{-\mu M_1 M_2} \quad (19)$$

will provide a massless neutralino. The particle content of this neutralino is given by:

$$N_{1i} = \left( \lambda \frac{v m_z}{\mu M_1} \cos 2\beta \sin \theta_w, -\lambda \frac{v m_z}{\mu M_2} \cos 2\beta \cos \theta_w, \right. \\ \left. -\lambda \frac{v}{\mu} \frac{0.6 m_z^2 \sin \beta - \mu M_1 \cos \beta}{\mu M_1}, -\lambda \frac{v}{\mu} \frac{0.6 m_z^2 \cos \beta - \mu M_1 \sin \beta}{\mu M_1}, 1 \right) \quad (20)$$

that is, in the limit when  $\lambda v/\mu \ll 1$ , it is mostly singlino. The coupling to the Z is given by:

$$O_{11}^L \simeq \left( \frac{\lambda v}{\mu} \right)^2 \frac{(\mu M_1)^2 - (0.6 m_z^2)^2}{(\mu M_1)^2} \simeq \left( \frac{\lambda v}{\mu} \right)^2 \quad (21)$$

and with  $\mu$  of order 500 GeV and  $\lambda \simeq 0.3$ , the constraint coming from the invisible Z width is satisfied again. (Note that this implies that  $\kappa$  is quite small as well).

### 3.3 The $c$ terms

We have seen that bi-linear terms (1) which couple the bino (or, in the NMSSM case, the singlino) and the right handed neutrino are allowed by the requirement that supersymmetry is softly broken. However, these terms break usual  $R$  parity, which may not be desirable. In

order to forbid the usual RPV terms, we can introduce a new  $U(1)_R$  symmetry. Let us then consider the following generation independent assignment of  $U(1)_R$  charges to the MSSM and right handed neutrino superfields:

$$\begin{array}{llll}
Q & (3, 2, 1/6) : & q & D^c & (\bar{3}, 1, 1/3) : & 2 - 2q - u \\
U^c & (\bar{3}, 1, -2/3) : & u & L & (1, 2, -1/2) : & -2 + q + u \\
N & (1, 1, 0) : & 2 & E^c & (1, 1, 1) : & 4 - 2q - 2u \\
H_d & (1, 2, -1/2) : & q + u & H_u & (1, 2, 1/2) : & 2 - q - u \\
\theta & : & 1 & & & 
\end{array} \tag{22}$$

where the  $SU(3)_c \times SU(2)_L \times U(1)_Y$  quantum numbers of the particles are given in the brackets, and  $\theta$  is the Grassmann coordinate. We choose the  $U(1)_R$  charge of the Grassmann coordinate  $\theta$  to be unity. It is easy to check that this charge assignment forbids the usual R-parity breaking terms and allows all MSSM Yukawa couplings as well as the right handed neutrino  $\tilde{B}N$  coupling and the Higgs mixing  $\mu$  term. In our  $U(1)_R$  charge approach we assume that the right handed neutrino fields have  $U(1)_R$  charge equal to  $-1$ ; this means that the corresponding mass terms are generated through a heavy field VEV ( $\geq 10^{14}$  GeV). This field also breaks the  $U(1)_R$  symmetry at the high scale.

In the NMSSM case, when the massless neutralino is mostly singlino, the relevant coupling with the right handed neutrino is:

$$c_i \tilde{S} N_i \tag{23}$$

This coupling also breaks usual  $R$  parity. But we can choose the following  $U(1)_R$  charge assignments:

$$\begin{array}{llll}
Q & (3, 2, 1/6) : & q & D^c & (\bar{3}, 1, 1/3) : & \frac{8}{3} - 2q - u \\
U^c & (\bar{3}, 1, -2/3) : & u & L & (1, 2, -1/2) : & -\frac{10}{3} + q + u \\
N & (1, 1, 0) : & \frac{10}{3} & E^c & (1, 1, 1) : & 6 - 2q - 2u \\
H_d & (1, 2, -1/2) : & -\frac{2}{3} + q + u & H_u & (1, 2, 1/2) : & 2 - q - u \\
\theta & : & 1 & S & & \frac{2}{3}
\end{array} \tag{24}$$

and in this case only the (23) couplings are allowed from general the RPV term.

## 4 Implications for neutrino sector

In this section we will consider the implications of a sterile neutrino coming from supersymmetry for neutrino physics.

### 4.1 LSND result and massless neutralino

As was mentioned briefly in sect. II, a massless neutralino seems ideally suited to explain the LSND experiment results. Since the magnitude of  $c$  couplings is given by the SUSY breaking scale, we can expect the mass of the fourth neutrino to be of order  $c^2/m_D^2 \simeq 100$  times larger than the mass of the heaviest SM neutrino. This puts it in the eV range, which is the right value needed to account for LSND result.



Moreover, LSND data [8] and constraints from short baseline experiments [16, 17] requires that the admixture of  $\nu_e$  and  $\nu_\mu$  in the fourth neutrino (let's call it  $\nu_0$ ) has to be of order of 0.1. In terms of the elements of the rotation matrix  $U_{\alpha i}$  ( $\alpha$  stands for  $\chi_1^0, \nu_e, \nu_\mu, \nu_\tau$ ) which diagonalizes the  $(M_\nu)_{\alpha\beta}$  4x4 mass matrix (3), we need  $U_{e0}, U_{\mu 0} \sim 0.12 - 0.14$ . On the other hand, as long as  $M_\nu(\chi_1^0, \nu_i) \ll M_\nu(\chi_1^0, \chi_1^0)$  we can employ the see-saw approximation to decouple the fourth neutrino from the other three, and we will have:

$$U_{i0} \simeq \frac{M_\nu(\chi_1^0, \nu_i)}{M_\nu(\chi_1^0, \chi_1^0)} \simeq \frac{m_D}{c} \simeq \frac{1}{10}$$

which fits nicely the experimental requirements.

For purposes of illustration, we present here a particular realization of this situation. We can work in a basis where the Majorana mass matrix is diagonal; let's even assume that it is proportional to the identity matrix:  $M_M = I_{3 \times 3} m_M$ . Moreover, let's assume that there is no hierarchy between the soft SUSY breaking parameters:  $c_1 = c_2 = c_3 = c$ . For the Dirac mass matrix, take a symmetric form:

$$m_D = \begin{pmatrix} m_1 & a_3 & a_2 \\ a_3 & m_2 & a_1 \\ a_2 & a_1 & m_3 \end{pmatrix} \quad (25)$$

and see what constraints the LSND result imposes on the elements of this matrix.

First, we need  $U_{e0} \simeq U_{\mu 0}$ ; since in the see-saw approximation  $U_{i0} = M_\nu(\chi_1^0, \nu_i)/M_\nu(\chi_1^0, \chi_1^0)$ , this implies  $\sum_i (m_D)_{i1} = \sum_i (m_D)_{i2}$ , or:

$$m_1 + a_2 + a_3 = m_2 + a_1 + a_3. \quad (26)$$

Moreover, from atmospheric oscillations results and the CHOOZ constraints on  $\bar{\nu}_e$  disappearance [18] ( $\theta_{23} \simeq 45^\circ, \theta_{13}$  small), we know that  $|U_{\mu 3}| \simeq |U_{\tau 3}| \simeq 1/\sqrt{2}$  and  $|U_{e3}| \simeq 0$ , which implies  $|U_{03}| \simeq 0$ , too. From the orthogonality of the rotation matrix  $\sum_i U_{i0} U_{i3} = 0$ , we then get  $|U_{\mu 0}| \simeq |U_{\tau 0}|$ , or

$$m_2 + a_1 + a_3 = m_3 + a_1 + a_2. \quad (27)$$

After decoupling the sterile neutrino, the effective mass matrix for the three SM neutrinos will be:

$$(M'_\nu)_{ij} = \frac{(m_D m_D^T)_{ij}}{m_M} - \frac{(M_\nu)_{i0} (M_\nu)_{0j}}{(M_\nu)_{00}}. \quad (28)$$

Since the determinant of this matrix is zero, there are three possible textures which will explain the observed neutrino mass splitting and mixings (see, for example, [19]). We shall try to obtain the hierarchical form, where  $(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) = (0, \delta, M)$ , with  $\delta \ll M$ . Then the mass matrix should look like (32). Requiring that  $|(M'_\nu)_{12}| \simeq |(M'_\nu)_{13}| \simeq 0$  we get the following equations:

$$m_2 = m_3, \quad a_1 = 2m_1 - m_3. \quad (29)$$

Then:

$$M'_\nu = \frac{1}{m_M} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2(m_1 - m_3)^2 & -2(m_1 - m_3)^2 \\ 0 & -2(m_1 - m_3)^2 & 2(m_1 - m_3)^2 \end{pmatrix} \quad (30)$$

leads to maximal mixing for the atmospheric neutrinos, with the corresponding mass scale given by  $M = 4(m_1 - m_3)^2/m_M$ . The  $\theta_{13}$  angle is also zero. However this texture does not explain the solar oscillations. To account for these, we need corrections of order  $\delta/M$  to the texture (30), where  $\delta$  is the mass scale responsible for solar oscillations. We are therefore led to consider corrections of order  $\delta/M$  to Eqs. (26, 27, 29). It turns out that, when  $\theta_{13}$  is chosen to be zero, Eq. (27) and the first one of Eqs. (29) are protected by the requirement that  $\theta_{23} = 45^\circ$ . Breaking relation (26) will give a mass to the second neutrino state, which can account for the splitting necessary for solar neutrino oscillations. An interesting note is that if the second relation in (29) remains unchanged, then the solar mixing angle will be given by  $\tan^2 \theta_{12} = 0.5$ , which is very close to the best fit value for solar neutrino oscillations.

With these choices, the following neutrino Dirac mass matrix is obtained:

$$m_D = \begin{pmatrix} m_1 & m_1 + \frac{\delta}{\sqrt{2}} & m_1 + \frac{\delta}{\sqrt{2}} \\ & m_3 & 2m_1 - m_3 \\ & & m_3 \end{pmatrix}$$

(the matrix being symmetric). This texture gives rise the following neutrino masses:

$$\{m_{\nu_0}, m_{\nu_1}, m_{\nu_2}, m_{\nu_3}\} = \left\{ 3\frac{c^2 + 3m_1^2}{m_M}, 0, \delta + \mathcal{O}(\delta\frac{m_1^2}{c^2}), 4\frac{(m_1 - m_3)^2}{M_M} \right\}$$

and the 4-neutrino mixing matrix will be:

$$U_{\alpha i} = \begin{pmatrix} \frac{\frac{c^2}{c^2+3m_1^2}}{\frac{m_1 + \sqrt{2}/3}{c} \sqrt{\delta m_M}} & 0 & 0 & 0 \\ \frac{\frac{c}{m_1 + \sqrt{2}/6}}{\frac{c}{m_1 + \sqrt{2}/6} \sqrt{\delta m_M}} & \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ \frac{\frac{c}{m_1 + \sqrt{2}/6}}{\frac{c}{m_1 + \sqrt{2}/6} \sqrt{\delta m_M}} & \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & -\frac{1}{\sqrt{2}} \\ \frac{\frac{c}{m_1 + \sqrt{2}/6}}{\frac{c}{m_1 + \sqrt{2}/6} \sqrt{\delta m_M}} & \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(with corrections of order  $m_1/c$ ). Here  $m_1$  can be taken to be arbitrary (as long as it is smaller than  $c$ ), and  $m_3$  to be fixed by the mass of the third neutrino:  $m_3 = \pm\sqrt{M_M m_{\nu_3}}/2 + m_1$ .

We have obtained the hierarchical structure for neutrino masses. Taking  $m_{\nu_3} = M \simeq 5. \times 10^{-2}$  eV and  $m_{\nu_2} = \delta \simeq 7. \times 10^{-3}$  eV, the splitting between  $\nu_3$  and  $\nu_2$  accounts for atmospheric neutrino oscillations, while the splitting between  $\nu_2$  and  $\nu_1$  accounts for the solar neutrinos. The atmospheric mixing angle is  $45^\circ$ , the solar angle is large ( $\simeq 35^\circ$ ), and the  $\theta_{13}$  angle is zero. Choosing  $c = 1$  TeV for  $m_M = 10^{15}$  GeV, and  $m_1 = 0.11c$ , we obtain the mass of the fourth neutrino  $m_{\nu_0} \simeq 3$ eV, and the elements of the rotation matrix:

$$U_{e0} \simeq 0.14, \quad U_{\mu 0} = U_{\tau 0} \simeq 0.12,$$

which values can account for the LSND results.

## 4.2 Massless neutralino and large mixing

Another interesting question is if it is possible to explain the large mixing among the three SM flavor eigenstates through the coupling to the neutralino. To this purpose, let's assume that the neutrino Dirac mass matrix has a diagonal structure:

$$(m_D)_{\alpha\beta} = m_\alpha \delta_{\alpha\beta}$$

Let's, moreover, assume that  $M_\nu(\chi^0, \chi^0) = (c_1^2 + c_2^2 + c_3^2)/m_M$  is much larger than the neutrino masses  $m_\alpha^2/m_M$ . Then, we can decouple the fourth neutrino, and in the see-saw approximation the mass matrix for the three SM neutrinos will become:

$$(M_\nu)'_{\alpha\beta} \simeq \frac{m_\alpha m_\beta}{m_M} \left( \delta_{\alpha\beta} - \frac{c_\alpha c_\beta}{\bar{c}^2} \right)$$

with  $\bar{c}^2 = c_1^2 + c_2^2 + c_3^2$ . Note that this matrix has zero determinant, but it is not traceless. As a consequence, the only possible diagonal neutrino mass matrices which can be obtained by diagonalization are the hierarchical  $\text{diag}(0, \delta, M)$  structure and the inverted hierarchy case  $\text{diag}(M, M + \delta, 0)$ . Both of these require that

$$|(M'_\nu)_{12}| \simeq |(M'_\nu)_{13}| \simeq 0$$

$$|(M'_\nu)_{22}| \simeq |(M'_\nu)_{33}| \simeq |(M'_\nu)_{23}| \simeq M/2$$

with corrections to these equations of order  $\delta/M$ . A simple choice which satisfies the above equations is

$$(c_1, c_2, c_3) = (0, c, c) ; (m_1, m_2, m_3) = (0, \sqrt{Mm_M}, \sqrt{Mm_M}) \quad (31)$$

which leads to a neutrino mass matrix:

$$M'_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M/2 & -M/2 \\ 0 & -M/2 & M/2 \end{pmatrix}. \quad (32)$$

This is the hierarchical case, with  $m_{\nu_3} = M$  and the angles  $\theta_{23} = 45^\circ$ ,  $\theta_{13} = 0$ . Corrections of order  $\delta/M$  to Eq. (31):

$$(c_1, c_2, c_3) = c \left( \sqrt{\frac{4\delta}{3M}}, 1 - \frac{\delta}{3M}, 1 - \frac{\delta}{3M} \right) ; \quad (33)$$

$$(m_1, m_2, m_3) = (\sqrt{\delta m_M/3}, \sqrt{Mm_M}, \sqrt{Mm_M})$$

will provide a mass  $\delta$  for  $\nu_2$  and make the  $\theta_{12}$  angle  $\simeq 35^\circ$ .

Finally, we want to mention that using a sterile neutrino in order to explain large mixing in the active sector is not a new idea (see, for example [10]). However, while in [10] the couplings of the sterile neutrino with the three SM flavors is introduced in a somewhat *ad hoc* manner, in our model it arises naturally.

## 5 Conclusions

In this paper we have addressed two questions: is a massless neutralino allowed by experimental data? and can such a neutralino couple with the neutrino sector and explain the LSND result, and/or large mixing among the three active neutrino flavors? We answered both questions in the affirmative. By giving up the assumption of universal gaugino masses,

we can obtain a massless neutralino in the MSSM. This would require fine-tuning, but, since its couplings to the  $Z$  can be suppressed, it will still satisfy the LEP constraints on the invisible  $Z$  width. Conversely, we can obtain a massless neutralino in the framework of the NMSSM, in which case this particle will be mostly a singlino.

The coupling of the massless neutralino with the neutrino sector is achieved through the introduction of new soft SUSY breaking terms of the form  $c\tilde{B}N$ . Then the neutralino, which is identified with a sterile neutrino, will acquire a mass proportional to the square of the soft SUSY breaking scale ( $c^2/M_M \sim \text{eV}$ ), in contrast with the usual see-saw where the light neutrino mass is proportional to the up quark or charged lepton mass square. This makes it an ideal candidate for explaining the LSND experiment results. Moreover, the see-saw induced mixing of the neutralino with the three neutrinos is also consistent with constraints from short baseline experiments, while large enough to account for LSND. Large mixing among the three active neutrino flavors arises naturally in this model.

The weak point of this model is that fine tuning is required to obtain a massless neutralino. A promising candidate for a massless SUSY particle would be a fermion associated with spontaneous SUSY breaking, that is, a visible sector goldstino. However, this would require some more model building, and we leave it for another paper. Even so, we find the fact that a massless neutralino is experimentally allowed interesting. Also, the fact that one of the light neutrino masses can be connected to the soft SUSY breaking scale is another intriguing feature of this scenario.

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